Coloring Discrete Structures

Provided by TryEngineering -www.tryengineering.org

Lesson Focus

Is it true or false that *Discrete Structures* and *Discrete Mathematics* are the same thing? This is the kind of question that is asked in this field – or both fields if they are indeed different. Most Middle School students see a mix of discrete and *continuous math* without ever noticing the difference. This lesson introduces them to areas of mathematics that computer scientists use to do computational problems. Search techniques through discrete structures are illustrated through *graph traversal* and *graph coloring*.

Age Levels

Intended for 11 - 13 (US Middle School grades 6 - 8)
Can be used in lower High School (e.g. 9th grade)

Objectives

Introduce students to:

- the relationship between Discrete Structures and Discrete Mathematics.
- the difference between discrete and continuous phenomenon.
- how to answer a question with sets.
- how a discrete problem is solved through search: specifically graph coloring.

Anticipated Learner Outcomes

Students will be able to:

- explain the difference between continuous and discrete structures.
- discuss the difference in perspective of discrete math and discrete structures.
- form a problem statement as a logical proposition, a Venn diagram, and an adjacency graph.
- color a map with the least number of colors.

Alignment to Curriculum Frameworks

See attached curriculum alignment sheet.

Internet Connections

- https://youtu.be/WX0hnuniLpI (Continuous vs. Discrete)
- https://youtu.be/ANY7X-_wpNs (Map coloring)

Recommended Reading

- http://wiki.acm.org/cs2001/index.php?title=Discrete_structures
- http://www.cse.buffalo.edu/~rapaport/191/S09/whatisdiscmath.html
- http://www.purplemath.com/modules/venndiag4.htm
- https://en.wikipedia.org/wiki/Set_(mathematics)
- https://en.wikipedia.org/wiki/Intersection_(set_theory)

Optional Writing Activity

Is it true or false that Discrete Structures and Discrete Mathematics are the same thing? Defend your answer.

Coloring Discrete Structures
For Teachers:
Teacher Resources

◆Lesson Objectives
Introduce students to:
- the relationship between Discrete Structures and Discrete Mathematics.
- the difference between discrete and continuous phenomenon.
- the ways to form a question with relations, sets, logic, and graphs.
- how to solve a problem, specifically graph coloring using search techniques.

◆Materials
- A 10 foot length of string or yarn.
- Ten pieces of heavy paper in at least four colors, large enough for a student to individually stand on.
- Sufficient copies of the coloring page given below for each student to have at least one.
- Crayons, colored pencils, or markers so that each student can have five unique colors. (Drawing implements can be shared.)

◆Procedure
It is not at all clear what the difference is between discrete mathematics and discrete structures, and in some sense the difference doesn't matter. The important ideas in both fields (or in the single field that is both) are:
1. discrete problems are approached differently from continuous ones.
2. using set theory can help you solve a problem.
3. search with backtracking is often the fallback method for solving a discrete problem.

This lesson introduces all three ideas through active learning. Work through the activities yourself ahead of time to fully understand what answers your students might produce.

Continuous vs. Discrete – No Worksheet
1. Watch the Continuous vs. Discrete video.
2. While your students are watching, lay the paper out around the classroom. Obstacles like desks are ok. You are essentially laying out colored paths, and some can have dead-ends. Mark one piece as the start, and another as the goal. (Perhaps use black or white for these.)
3. Pose the discrete problem: "Using stepping stones, can you get between two points if you can't see all of the stones?" Explain that this is a discrete structures problem.
    a. Line up all of your students, or use a subset as volunteers.
    b. As each student starts on the path, assign her a color path to avoid. Tell them if they get stuck, they should just stop and wait. If someone else is on a square, they must wait for that person to move on.
    c. Give the last person sufficient time to get stuck or get to the destination.
    d. Ask your students why this might be thought of as a discrete problem.
4. Pose the continuous problem: "If you can only go half-way to somewhere, will you ever get there?"
    a. Ask for three volunteers.

b. Give two of the three volunteers one end of the string and spread them apart 10 feet (or 3 meters.)
c. Direct the third student (the traveler) to stand with the first student.
d. Direct the traveler to go half way. Continue directing the traveler to move another half distance until she can't get closer by taking a half step, because she is too close to the end destination and doesn't have room to move.
e. This next instruction is critical. Explain to your students that in a continuous world we can expand the distance between two things. By zooming in on the distance between the traveler and the destination, in our imaginations we can create a distance big enough for her to travel through. Do this by having the traveler return to the origin. Make sure students see what's happening here.
f. Repeat the process, explaining that there is an infinite amount of space between the origin and the destination.
g. Explain that this is the kind of problem that continuous mathematics studies. We have to believe, but can never tell, whether that infinite space in between the start and finish is predictable. Solicit answers to the question: will the traveler ever get there if she can only go half way with each step?

Which Set is it In?

To answer the controversy about whether it is discrete math or discrete structures, have your students use Worksheet 1 to place discrete topics in the math, structures, or intersection of the Venn diagram. The topics are listed on the 'Student Resource' page. Have a discussion about the question 'Is it true or false that Discrete Structures and Discrete Mathematics are the same thing?' Discuss how set theory can be used to address, but not necessarily definitively answer, a logic question.

Graph Theory through Map Coloring

1. Watch the Four Colour Theorem Video.Show them the images on last page of this teacher resource from https://en.wikipedia.org/wiki/Four_color_theorem. Either print the page, show the page online or go directly to the Wikipedia page.
2. Direct your students to choose four different colored crayons (pencils, markers). It's ok to share colors. Consider doing this exercise in pairs. Talking through the solution with someone is helpful.
3. Have them try to color the picture with four colors as shown in the video. Instruct them to 'think ahead'. Suggest that they might want to put down just a thin line of color in each region so that if they get stuck they can *backtrack* if necessary. Have extra copies handy for those who really do need to start over.
4. When everyone has completed the drawing, or is stuck and giving up. Ask them as a group whether they all ended up with the same solution. Encourage them to realize that the colors were arbitrary, each had colors 1, 2, 3, 4. One person's red might have been another's green.
5. Direct them to compare their solution with someone next to them. On the back of their sheet have them record the names of the students whose solution is the same, and whose is different. Suggest that they shouldn't have to compare directly with everyone in the room, but can use these names later as a faster way to see whose is alike or different.
6. Once everyone is compared. Ask if anyone found differences.
    a. If not, ask them whether that means there is definitely only one solution. (There isn't definitively – just because they all have the same doesn't mean there couldn't be another solution out there that was missed.)

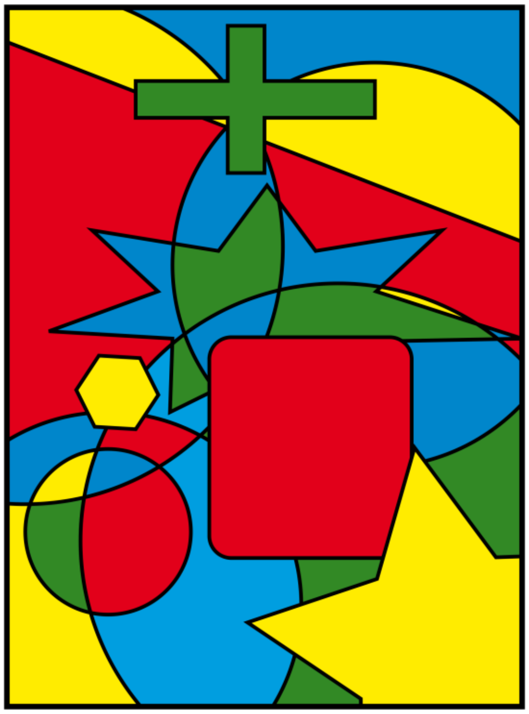
    b. If there were differences, call up two students who had different solutions, and ask them to stand at opposite sides of the room.
    c. Ask the rest of the class to join one of those students if they have that student's name on their sheet.
    d. If anyone remains seated, call up one of them and have them stand apart from the others. Ask the remaining seated students to join the new solution if that student's name is on their sheet. Do this until everyone is standing. Clearly there is more than one solution.
7. There might have been a flaw in the procedure in step 6. It's possible that students got it wrong – that two solutions that appeared to be the same were different, or that two different, were the same. Ask everyone to check whether they have the name of someone in a different group on their sheet. If they do, and if you have time, check their answers and regroup. If you don't have time, simply use this as an object lesson, that a 'proof' is only as good as the information coming in. But if this happens to end on a good note for this to happen. You still know that there is more than one solution, but you still don't know if the solution is unique. These are the kinds of problems solved with discrete structures!

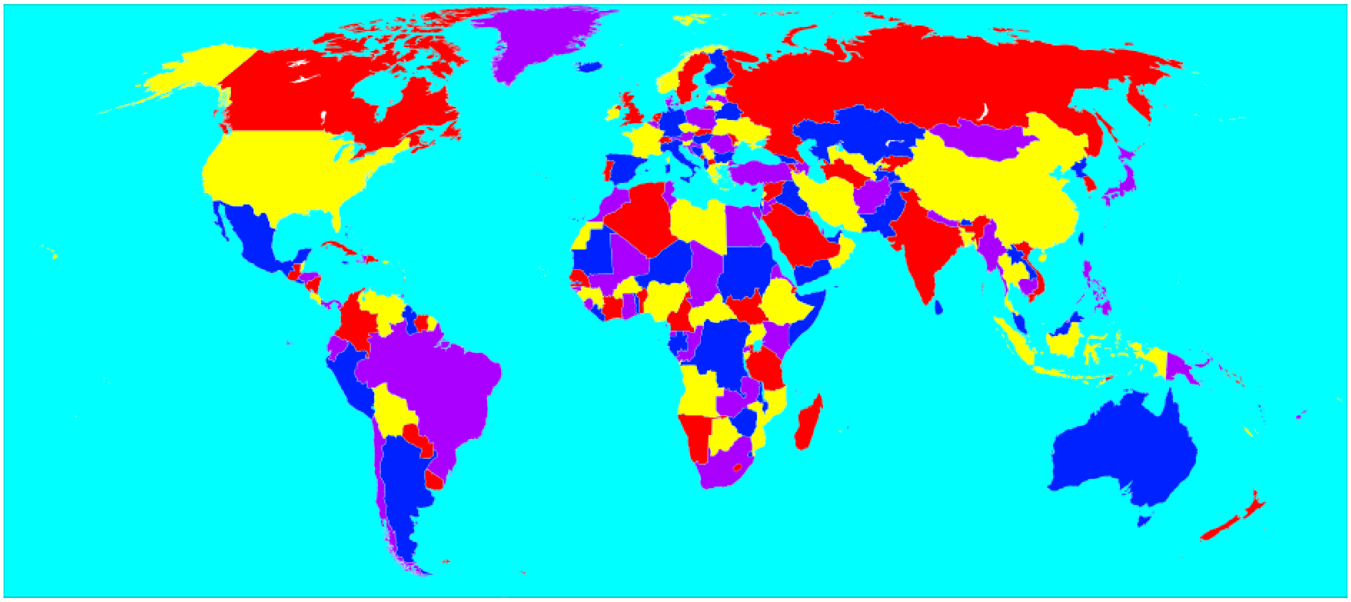
◆Time Needed

- 2 sessions, at most 1 hour each. You should be able to do the first two activities in a single session, and the coloring problem in the third. If you move more quickly, you can always have them create their own drawing and challenge a friend to color it in.

Some classic four color examples

https://en.wikipedia.org/wiki/Four_color_theorem





Coloring Discrete Structures
Student Resource:

Discrete Mathematics Topics
(https://en.wikipedia.org/wiki/Discrete_mathematics)

Theoretical computer science
Information theory
Logic
Set theory
Combinatorics
Graph theory
Probability
Number theory
Algebra
Calculus of finite differences
Geometry
Topology
Operations research
Game theory, decision theory

Discrete Structures Topics
(http://wiki.acm.org/cs2001/index.php?title=Discrete_structures)

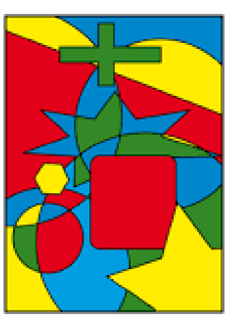
Sets
Logic
Boolean Algebra
Proof Techniques
Counting
Graphs and trees
Discrete Probability
Boolean algebra
Set theory

From Meriam-Webster.com:

Continuous:	continuing without stopping; happening or exiting without a break or interruption
Discrete:	separate and different from each other
Map:	a picture or chart that shows the rivers, mountains, streets, etc. in a particular area; a picture or chart that shows the different parts of something
Set (noun, def 2):	a group of things of the same kind that belong or are used together
Subset:	a group of things, people, etc. that is part of a larger group

From Wikipedia.org

Set:	In mathematics, a set is a collection of distinct objects, considered as an object in its own right. For example, the numbers 2, 4, 6 are distinct objects when considered separately, but when they are considered collectively they form a single set of size three.
Set intersection:	(paraphrased) In mathematics, the intersection of two sets A and B is the set that contains all of the elements A that also belong in B, but no others.



In mathematics, the **four color theorem**, or the **four color map theorem**, states that, given any separation of a plane into contiguous regions, producing a figure called a map, no more than **four colors** are required to **color** the regions of the map so that no two adjacent regions have the same **color**.

.

Coloring Discrete Structures
Student Worksheet 1:

Is it true or false that Discrete Structures and Discrete Mathematics are the same thing?

The question above poses a question in a way that we might be able to answer it with logic. It is a very simple research statement. To answer it definitively, you would have to do some research on the Internet to see whether reliable sources make a distinction or not, and who refers to structures and who refers to mathematics. A place to start to just address the question is to look at the topics considered in each. Using set theory, if the two *sets* of topics are equal, then they are the same.

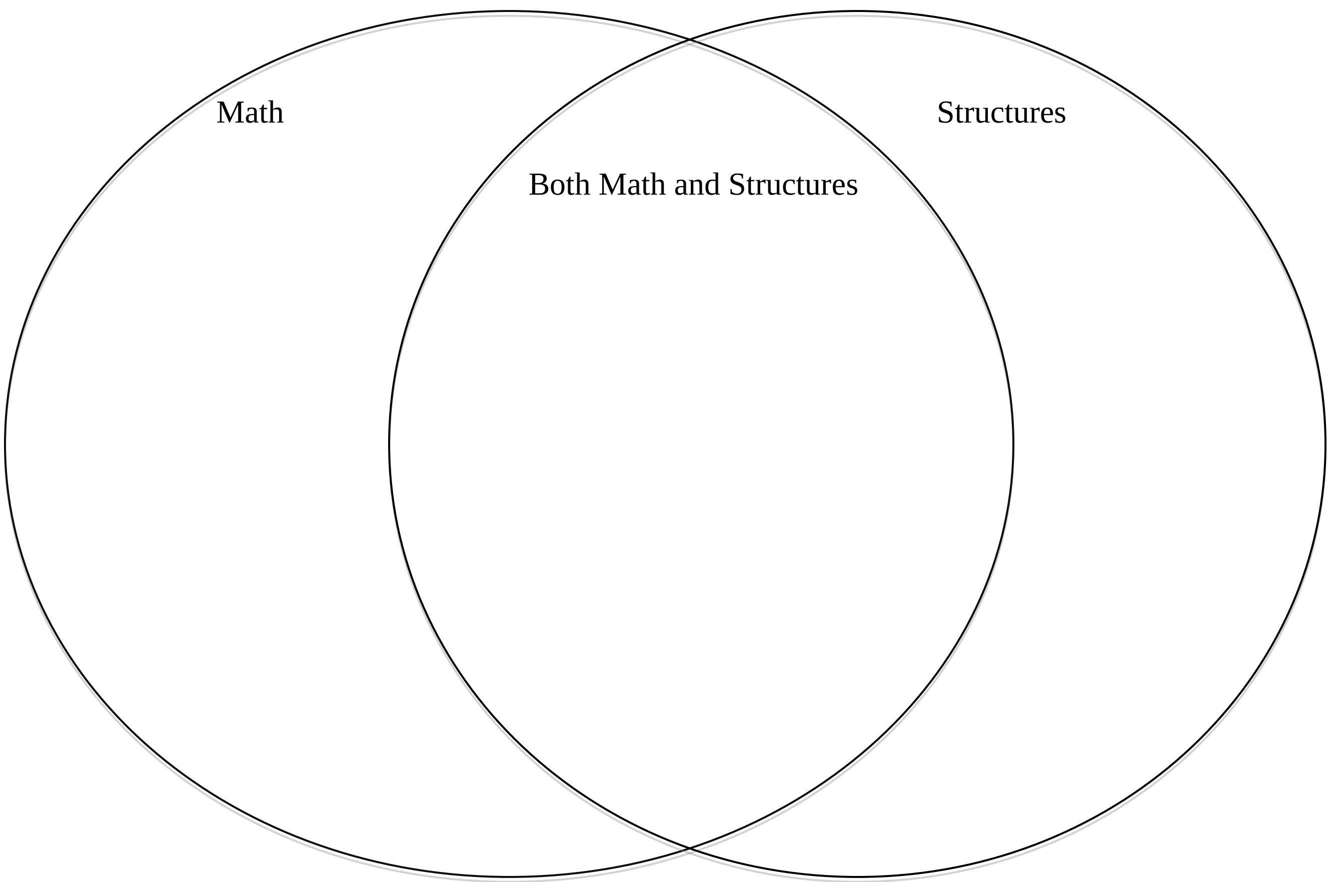
With a partner, use the lists of topics from the Student Resource Sheet to decide whether a topic is part of Discrete Mathematics and which are part of Discrete Structures.

A Venn Diagram gives you a way to view the elements in a set. Use the Venn diagram below and the definitions on the Student Resource Sheet to place topics in the correct parts of the diagram.

Using the definitions on the Student Resource Sheet:

Are all topics in the intersection (are the sets equivalent)? ________________

Is one set a subset of the other? ________________



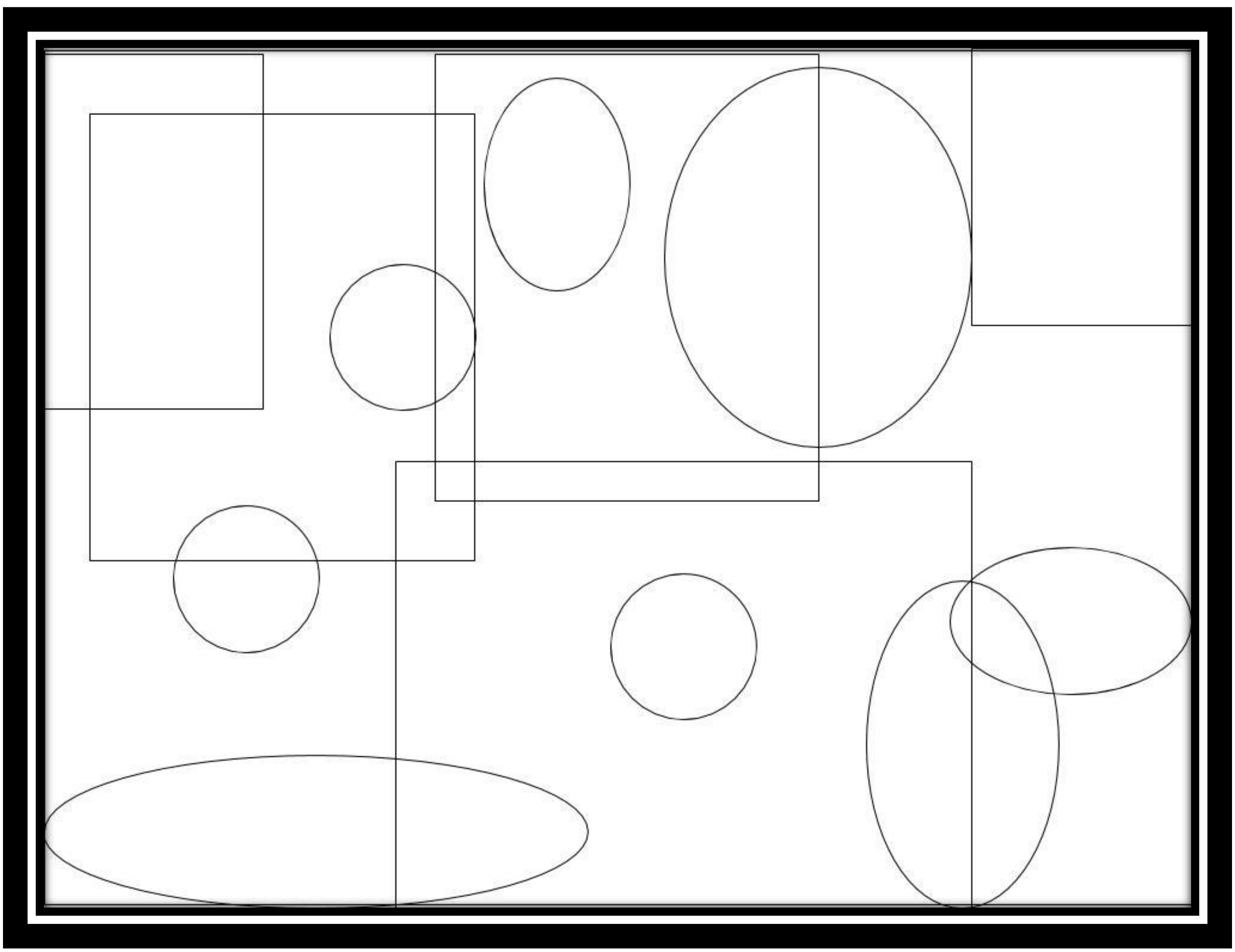
Coloring Discrete Structures
Student Worksheet 2:

Can this picture be colored in with just for colors?

The rule is that the same color can touch at a corner but it can't be used in two adjacent areas.

Is there more than one way to color this picture with just four colors? You will answer this question as a class!

Note to readers of the draft. The general outline show here will be used, however the graphic needs to be edited so that the tiny areas are not shown. This image ***can*** be four colored in more than one unique way. Proof by two pictures ☺



For Teachers:
Alignment to Curriculum Frameworks

Note: All lesson plans in this series are aligned to the Computer Science Teachers Association K-12 Computer Science Standards, and if applicable also the U.S. Common Core State Standards for Mathematics, the U.S. National Council of Teachers of Mathematics' Principles and Standards for School Mathematics, the International Technology Education Association's Standards for Technological Literacy, the U.S. National Science Education Standards and the U.S. Next Generation Science Standards.

◆National Science Education Standards Grades 5-8 (ages 10-14)

CONTENT STANDARD E: Science and Technology

As a result of activities, all students should develop

- ✦ Understandings about science and technology

◆National Science Education Standards Grades 9-12 (ages 14-18)

CONTENT STANDARD E: Science and Technology

As a result of activities, all students should develop

- ✦ Understandings about science and technology

◆Next Generation Science Standards & Practices Gr.5-8 (ages 10-14)

Practice 5: Using Mathematics and Computational Thinking

- ✦ Use mathematical representations to describe and/or support scientific conclusions and design solutions.

◆ Principles and Standards for School Mathematics (all ages)

Problem Solving Standards

- ✦ Solve problems that arise in mathematics and other contexts

Connections

- ✦ Recognize and apply mathematics in contexts outside of mathematics

Representations

- ✦ Use representations to model and interpret physical, social, and mathematical phenomena

◆Common Core State Practices & Standards for School Mathematics (all ages)

- ✦ CCSS.MATH.PRACTICE.MP1 Make sense of problems and persevere in solving them.
- ✦ CCSS.MATH.PRACTICE.MP4 Model with mathematics.
- ✦ CCSS.MATH.PRACTICE.MP7 Look for and make use of structure.

◆ Standards for Technological Literacy - All Ages

Nature of Technology

- ✦ Standard 2: Students will develop an understanding of the core concepts of technology

Coloring Discrete Structures

For Teachers:
Alignment to Curriculum Frameworks

◆CSTA K-12 Computer Science Standards Grades 6-9 (ages 11-14)

2 Level 2: Computer Science and Community (L2)

- Computational Thinking: (CT)

  8. Use visual representations of problem states, structures, and data (e.g., graphs, charts, network diagrams, flowcharts).

  14. Examine connections between elements of mathematics and computer science including binary numbers, logic, sets and functions.

◆CSTA K-12 Computer Science Standards Grades 9-12 (ages 14-18)

5.3 Level 3: Applying Concepts and Creating Real-World Solutions (L3)

5.3.A *Computer Science in the Modern World (MW)*

- Computing Practice and Programming (CPP)

  12. Describe how mathematical and statistical functions, sets, and logic are used in computation.